Math 241 Sample Problems for Exam 2

Question 1 Use Lagrange multipliers to find the maximum and minimum values of $f(x, y, z) = x - 2y + 5z$ on the sphere $x^2 + y^2 + z^2 = 30$.

Question 2 Evaluate the following double integral:

$$\int_0^2 \int_{y/2}^1 ye^{x^3}\, dx\, dy$$

Question 3 Find the volume of the solid in space which lies below the surface $z = 3 + \cos y$ and above the region in the xy–plane bounded by the curves $x = \pi$, $y = 0$, and $y = 2x$ by evaluating an appropriate double integral.

Question 4 Let R be the solid region bounded by the planes $x = 0$, $y = 0$, $z = 2$, and the paraboloid $z = x^2 + y^2$, in the first octant. Compute $\iiint_R x\, dV$.

Question 5 Find the volume determined by $z \leq 6 - x^2 - y^2$ and $z \geq \sqrt{x^2 + y^2}$.

Question 6 Convert the integral $\displaystyle\int_{-\sqrt{2}}^{\sqrt{2}} \int_{-\sqrt{2-y^2}}^{\sqrt{2-y^2}} \int_{\sqrt{x^2+y^2}}^{\sqrt{4-x^2-y^2}} z^2\, dz\, dx\, dy$ to spherical coordinates. Don't evaluate it.

Question 7 Evaluate $\displaystyle\iint_R \exp\left(\frac{y-x}{y+x}\right) dA$ where R is the triangle with vertices $(0, 0)$, $(1, 0)$, $(0, 1)$, by using the change of variables $x = \frac{1}{2}(v - u)$, $y = \frac{1}{2}(u + v)$.

Question 8
a) Evaluate the line integral $\int_{\mathbf{c}} x^2 y\, dx + xy^3\, dy$, where $\mathbf{c}$ consists of the line segments from $(0, 0)$ to $(3, 3)$ and from $(3, 3)$ to $(0, 3)$.
b) Evaluate the line integral $\int_{\mathbf{c}} (e^y + ye^x)\, dx + (e^x + xe^y)\, dy$, where $\mathbf{c}$ is the part of the graph $y = \ln 6x$ joining $(1/6, 0)$ to $(1/2, \ln 3)$.

Question 9 Find the area of the surface that is part of the sphere $x^2 + y^2 + z^2 = 4z$ that lies inside the paraboloid $z = x^2 + y^2$.